TWO TALKS BY

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1: Herrmann's work on the arithmetical complexity of infinite chains and antichains in computable partial orderings

It is a direct consequence of Ramsey's theorem for pairs that every partial ordering of ω has either an infinite chain or an infinite antichain. Herrmann [1] studied where such infinite chains and antichains arise in the arithmetical hierarchy when the given partial ordering is computable. He showed in [1, Theorem 1.1] that every computable partial ordering of ω has either an infinite Δ^0_2 chain or an infinite Π^0_2 antichain. (See also [2, Theorem 2.2] and the remark just after it.) In [1, Theorem 3.1] Herrmann gave an elegant proof that this is best possible in the sense that there is a computable partial ordering $<_P$ of ω which has no infinite Σ^0_2 chains or antichains. I will present an exposition of this proof. Herrmann also obtained a "quasiuniqueness" result for such partial orderings. Let $\mathcal{C}$ be the class of all computable *linear* orderings $<_L$ of ω which are extensions of computable partial orderings $<_P$ of ω such that $<_P$ has no infinite Σ^0_2 chains or antichains. Herrmann [1, Corollary 4.2] showed that any two orderings in $\mathcal{C}$ are isomorphic, and in fact his argument shows that all orderings in $\mathcal{C}$ are of order-type $\omega + (\omega^* + \omega)\eta + \omega^*$, where η is the order type of the rationals. If time permits, I will also discuss the proof of this result, including a correction due to Joseph Mileti.

References

[1] E. Herrmann, Infinite chains and antichains in computable partial orderings, J. Symbolic Logic 66 (2001), 923–934.

[2] C. Jockusch, Ramsey's theorem and recursion theory, J. Symbolic Logic 37 (1972), 268–280.

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2: The logical strength and effective content of the thin set theorem and the free set theorem.

I will be discussing joint work with Peter Cholak, Mariagnese Giusto and Jeffry Hirst. Let $[A]^k$ denote the set of all k-element subsets of the set A. The *thin set theorem*, due to Harvey Friedman, (see [2] or [1]), asserts that for every $k \in \omega$ and every function $f : [\omega]^k \to \omega$ there is an infinite set $A \subseteq \omega$ such that $f([A]^k) \neq \omega$. The *free set theorem*, also due to Harvey Friedman (see [2] or [1]), asserts that for every k and every function $f : [\omega]^k \to \omega$ there is an infinite set $A \subseteq \omega$ such that, for all $D \in [A]^k$, if $f(D) \in A$, then $f(D) \in D$. We partially analyze the strength of these two results (and especially their restrictions to fixed k) in the sense of reverse mathematics. We also study the effective content of these results for fixed k, and compare them with corresponding results for Ramsey's Theorem [3]. For example, we show that if $f : [A]^k \to \omega$ is computable, then the set A asserted to exist in the free set theorem can be chosen to be Π^0_k, and this is best possible with respect to the arithmetical hierarchy for $k > 1$. The analogous result holds for the thin set theorem.

References

[1] Peter A. Cholak, Mariagnese Giusto, Jeffry L. Hirst, and Carl G. Jockusch, Jr.,*Free sets and reverse mathematics,* submitted for publication (available online at www.math.uiuc.edu/~jockusch/ under "Online Articles").

[2] Harvey Friedman and Stephen G. Simpson, *Issues and problems in reverse mathematics,* **Computability theory and its applications (Boulder, CO, 1999)**, Amer. Math. Soc., Providence, RI, 2000, 127–144.

[3] Carl G. Jockusch, Jr., *Ramsey's Theorem and recursion theory,* J. Symbolic Logic **37** (1972), 268–280.